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AUTHOR Kirby, Peggy C.; Abernathy, Mari W.
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ABSTRACT

Canonical correlation analysis is the best technique to employ when the research problem has multiple predictor and multiple criterion (outcome) variables, which is usually the case in the "real" world of education. A hypothetical data set is presented to illustrate how this particular multivariate method can be used to detect effects of interaction between variables in a predictor variable set and the composite of criterion variables. In the simulated data, IQ and socioeconomic status (SES) are the variables used to predict an achievement test score and grade-point average (GPA). To test for interactions, a dummy predictor variable is created as the product of SES and IQ. This product variable is a significant predictor itself, and improves the explanatory power of the canonical functions. The three-variable model (with product variable) greatly reduces the Wilks' Lambda over the two-variable model. Furthermore, the three-variable model more than doubles the explained variance for GPA. The preference for this technique over MANOVA is based on the ability of canonical correlation to detect interactions without having to transform continuous data to arbitary nominal levels, permitting more accurate generalization to multivariate reality. An extensive (40 pages) Statistical Analysis System (SAS) printout is attached. (Author/LPG)

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The Multivariate Reality of Educational Research: Detecting Interaction Effects Using Canonical Correlation Analysis

Peggy C. Kirby

University of New Orleans 70148

Mari W. Abernathy

University of New Orleans 70148
and
Louisiana State University Medical Center 70112

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ABSTRACT

The "real" world of education is multifaceted and complex. The analytic methods used in educational evaluation and research should honor this reality. Canonical correlation analysis is a multivariate method which allows the investigator to extrapolate the most information from the available data when there are multiple predictor and multiple criterion (outcome) variables. A hypothetical data set is presented to illustrate how canonical correlation analysis can be used to detect effects of interaction between variables in a predictor variable set and the composite of the criterion variables. A Statistical Analysis System (SAS) printout is attached for reference.

The Multivariate "Real World" of Educational Research

The use of multivariate methods in the evaluation of educational effectiveness is essential to the researcher who views the educational process as a multifaceted and complex system of variables. Hopkins (1980) noted that research questions having to do with the education of human beings require the use of statistical methods which are able to analyze simultaneously many interrelated variables. McMillan and Schumacher (1984, pp. 269-270) share this view:

> Social scientists have realized for many years that human behavior can be understood only by examining many variables at the same time, not by dealing with one variable in one study, another variable in a second study, and so forth... These (univariate) procedures have failed to reflect our current emphasis on the multiplicity of factors in human behavior... In the reality of complex social situations the researcher needs to examine many variables simultaneously.

The claim by Kerlinger (1973) that multivariate methods "mirror" educational reality better than other methods has been substantiated. Indeed, Thompson (1986b) has demonstrated that multivariate methods, and not univariate, clearly are the best where the researcher is concerned with honoring the reality that education involves multiple causes and multiple effects. He noted that the full network of variables usually operates simultaneously. This effect of interaction, when it occurs, cannot be overlooked if the researcher desires to utilize all of the information contained in the data and make accurate generalizations.

Canonical Correlation Analysis as the Most General Linear Model

Canonical Correlation Analysis (Hotelling, 1935) is a multivariate method which has been shown to be the most general case of the general linear model (Baggaley, 1981; Fornell, 1978). That is to say "virtually all the commonly encountered parametric tests of significance can be treated as special cases of canonical correlation analysis" (Knapp, 1978, p. 410).

According to Afifi and Clark (1984, p. 361), "the technique of canonical correlation analysis is best understood by considering it as an extension of multiple regression and correlation analysis". It is the most appropriate analytic technique when each of the variable sets has more than one element--that is, two or more independent or predictor, variables and two or more dependent, or criterion variables (Thompson, 1984). Afifi and Clark (1984, p. 362, emphases in original) explain:

> The technique consists of finding several linear combinations of the X [independent, predictor] variables and the same number of linear cominbations of the Y [dependent, criterion] variables in such a way that these linear combinations best express the correlations between the two sets. Those linear combinations are called canonical variables, and the correlations between corresponding parts of canonical variables are called canonical correlations.

Canonical correlation analysis is not new. The technique was developed over fifty years ago (Hotelling, 1935). However, researchers have found use of the technique prohibitive due to the intricate mathematical computations involved (Baggaley, 1981). Use of the technique is gaining in popularity with the availability of sophisticated statistical computer packages. For an indepth discussion of the canonical correlation technique, including some

underlying mathematics, the reader is referred to Thompson's Canonical Correlation Analysis: Uses and Interpretations (1984).

Detecting Interaction Effects Using Canonical Correlation Analysis

Interaction of independent variables is the effect of two independent variables operating together on the dependent variable as distinguished from their separate individual effects (Huck, Cormier, & Bounds, 1974, p.78). Since multivariate techniques are designed to consider how variables operate together, methods are available for detecting additive effects of additional variables as well as multiplicative (interactive) effects.

In the multivariate analysis of variance (MANOVA) case, a significance test of interactions is easily obtained. Unfortunately, interactions are not as easily detected in the canonical case. Of course, interaction is implied whenever structure coefficients do not equal function coefficients (Thompson, 1984, p. 23). Preference for canonical techniques is often desirable in order to maximize all available data; i.e., to eliminate arbitrary transformation of continuous data to nominal levels (Thompson, 1962). When an interaction effect is hypothesized, a coded variable can be added to the canonical model to test the assumption that the resulting canonical R-squared is increased (Afifi & Clark, 1984, pp.225-227). This procedure can be utilized when interactions are suspected either between independent variables or between dependent variables since there is no distinction between the two sets in canonical correlation analysis. This is a clear advantage over MANOVA which tests only interaction effects of independent variables on the criterion variables.

To illustrate this coding technique, a simple data set (p. 10) was

was created that would reveal a strong interaction effect of two independent variables on one of two criterion variables. It is supposed that a researcher is interested in predicting a measure of high school success based on student IQ and socio-economic status (SES). Lacking a single variable that operationally defines school success, the researcher chooses two measures: score on a standardized achievement test and grade point average (GPA) for the freshman year.

Theoretically, IQ should greatly influence the achievement test score and GPA variables, but the researcher has reason to believe that GPA is also affected by teachers' perceptions of student socio-economic status. In this example, high IQ students scored higher overall on the achievement test than did low IQ students, but high IQ wealthy students had GPA's very similar to low IQ wealthy students. High IQ poorer students had the highest GPA's while low IQ poorer students had the lowest GPA's. IQ above 100 is coded high, below 100 is coded low. All IQ scores are either greater than or equal to 110 or less than or equal to 85. Again to make the interaction obvious, mid-range scores were avoided. Students whose parents earned $40,000 or more were coded as high SES and students whose parents earned $25,000 or below were coded low SES.

The computer program used in this example is the Statistical Analysis System (SAS) package. A computer printout is attached for reference. The reader may wish to replicate the findings using other hypothetical data or actual data.

Tables of means by level of IQ and SES are presented on page 16. MANOVA results (pp.17-22) reveal that the omnibus F's for IQ and the interaction of IQ and SES are significant ($p < .0001$) whereas the F for SES is not. Univariate F's indicate that the interaction is only significant

($p < .0001$) for GPA, not for achievement test score (pp. 14 and 15). Results of a canonical correlation using Z-scores for all variables (confirmed on p. 11) begin on page 23. Note that the canonical R-squared of the first function for this model is .85. That is, 85% of the variance in the criterion composite is explained by the first predictor composite.

Standardized function coefficients (p. 24) reveal that the first canonical variate of the first set consists largely of IQ, and the first criterion composite consists mainly of achievement score. Thus, the first function does little more than the simple bivariate correlation between IQ and achievement score. This simple correlation is .90 (see p. 12) as compared to the adjusted canonical correlation coefficient for the first function, .91. The second function is not statistically significant. The second canonical variate for the first variable set explains little more than 1% of the second criterion composite.

Although the canonical R-squared of .84 for the first function is very high, the interaction of IQ and SES may further improve the model. The additive effects of the two variables have been considered but their multiplicative effects have not. To test the interaction, a dummy variable is created by multiplying the Z-score forms of the original predictor variables. This product variable alone produces a significant ($p < .001$) squared canonical correlation of .68 (p. 26). In conjunction with the original two variables, two significant canonical functions are extracted (p. 29). The first consists mainly of IQ score while the second improves the model through the contribution of the product or interaction variable.

Note the differences in Wilk's lambda for the 2-predictor variable model and the model that includes the product variable as a third predictor. Lambda decreases from .16 (p. 23) to .04 (p.29). Also note that the

correlations between GPA and the two predictor composites are .60 and .08 for the model that does not consider interactions (p. 25). Thus, $(.6022)^2 = (.0819)^2$ or only 36.9% of the GPA variance is explained by the two predictor composites. On the other hand, the model that contains the product variable has GPA-predictor composite correlations of .85 and .12. Squaring and adding yields an explained variance of 84.3%, more than twice the squared index coefficient of the model that considers only additive effects of independent variables.

These results are replicated by considering a canonical model involving only the original predictor variables and their product in the first set and GPA alone in the second (see pp. 32-40). With the original two independent variables, the R_c-squared is 36.9%, the same as the sum of the squared correlations between GPA and the predictor composites from page 15. Adding the product variable increases R_c-squared to 84.3%, as noted above.

Stability of results was investigated using the RELATE computer program (Veldman, 1967) to rotate standardized canonical function coefficients for a subset of the original sample to a position of best fit with standardized canonical function coefficients for a holdout sample (see pp. 41-50). The cosines of the angles between factors are correlation coefficients. The correlation between factors for the first sample with corresponding factors for the second sample is .795, a moderate invariance coefficient although there is also a moderate degree of correlation between factors of the same set (see invariance log). Test R's all exceed .94 meaning that all the variables fit well into a common factor space after rotation.

In conclusion, canonical correlation can be useful in detecting the effects of interaction of two independent variables on a criterion variable or a composite of criterion variables. The product of the two predictor variables

will improve the model if an interaction effect does exist. In fact, the degree of improvement can be evaluated using canonical commonality analysis (Thompson & Miller, 1985). In the example provided, IQ and SES had a strong interactive effect on student GPA although SES had little predictive power of its own and could actually have been eliminated from the model.

Summary

Multivariate (versus univariate) statistical methods are the most appropriate when studying complex phenomenon. Canonical correlation analysis is the best technique to employ when the research problem has multiple predictor and multiple criterion variables, which is usually the case in the "real" world of education. Further, canonical correlation analysis has the ability to detect effects of interaction between the variables in the predictor variable set and/or between the variables in the criterion variable set. The preference for this technique over MANOVA is based on the ability of canonical correlation to detect interactions without having to transform continuous data to arbitrary nominal levels. Thus, the researcher maximizes the information contained in the raw data and is able to make accurate generalizations to multivariate reality.

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RAW DATA

IQ LEVEL	IQ	SES LEVEL	SES	ACHIEVEMENT	GPA	INVAR GROUP
L	060	L	20	29	1.0	1
L	065	L	25	32	0.8	2
L	080	L	15	35	0.7	1
L	075	L	10	28	0.5	2
L	085	H	50	35	2.0	1
L	060	H	55	30	2.5	2
L	070	H	60	12	2.2	1
L	072	H	45	18	2.3	2
H	112	L	20	70	3.6	1
H	110	L	25	65	3.1	2
H	126	L	12	75	3.5	1
H	118	L	11	80	3.4	2
H	110	H	50	85	2.0	1
H	120	H	65	56	2.1	2
H	125	H	60	90	2.2	1
H	115	H	40	60	2.0	2

DESCRIPTIVE STATISTICS

SAS 11:09 MONDAY, DECEMBER 8, 1986 1

VARIABLE	N	MEAN	STANDARD DEVIATION	MINIMUM VALUE	MAXIMUM VALUE	STD ERROR OF MEAN	SUM	VARIANCE	C.V.
IQ	16	93.93750000	24.98657973	60.00000000	126.00000000	6.24664493	1503.0000000	624.32916667	26.599
SES	16	35.18750000	19.84008989	10.00000000	65.00000000	4.96002247	563.0000000	393.62916667	56.384
ACHIEVE	16	50.00000000	25.40078739	12.00000000	90.00000000	6.35019685	800.0000000	645.20000000	50.802
GPA	16	2.11875000	0.98537218	0.50000000	3.60000000	0.24634305	33.9000000	0.97095833	46.507
ZIQ	16	-0.00010004	0.99986313	-1.35814326	1.28291317	0.24996578	-0.0016006	0.99972629	-99999.000
ZSES	16	-0.00012601	1.00000453	-1.26965726	1.50252016	0.25000113	-0.0020161	1.00000906	-99999.000
ZACHIEVE	16	-0.00000000	1.00003100	-1.49606299	1.57480315	0.25000775	-0.0000000	1.00006200	-99999.000
ZGPA	16	-0.00025381	1.00037785	-1.64365482	1.50355330	0.25009446	-0.0040609	1.00075584	-99999.000
PROD	16	0.00985621	1.01324011	-1.49953409	1.56685376	0.25331003	0.1576994	1.02665553	10280.220

SIMPLE BIVARIATE CORRELATIONS

VARIABLE	N	MEAN	STD DEV	SUM	MINIMUM	MAXIMUM
IQ	16	93.93750000	24.98657973	1503.00000000	60.00000000	126.00000000
SES	16	35.18750000	19.84008989	563.00000000	10.00000000	65.00000000
PROD	16	0.00985621	1.01324011	0.15769937	-1.49953409	1.56685376
ACHIEVE	16	50.00000000	25.40078739	800.00000000	12.00000000	90.00000000
GPA	16	2.11875000	0.98537218	33.90000000	0.50000000	3.60000000

PEARSON CORRELATION COEFFICIENTS / PROB > |R| UNDER H0:RHO=0 / N = 16

	IQ	SES	PROD	ACHIEVE	GPA
IQ	1.00000	0.01051	0.04602	0.90135	0.60658
	0.0000	0.9692	0.8656	0.0001	0.0127
SES	0.01051	1.00000	0.15235	-0.07144	0.04448
	0.9692	0.0000	0.5733	0.7926	0.8701
PROD	0.04602	0.15235	1.00000	0.07770	-0.64603
	0.8656	0.5733	0.0000	0.7749	0.0069
ACHIEVE	0.90135	-0.07144	0.07770	1.00000	0.54470
	0.0001	0.7926	0.7749	0.0000	0.0291
GPA	0.60658	0.04448	-0.64603	0.54470	1.00000
	0.0127	0.8701	0.0069	0.0291	0.0000

MANOVA: CLASS LEVEL INFORMATION

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
IQM	2	H L
SESM	2	H L

NUMBER OF OBSERVATIONS IN DATA SET = 16

ANOVA RESULTS - DEPENDENT VARIABLE: ACHIEVE

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: ACHIEVE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	3	8295.50000000	2765.16666667	24.00	0.0001	0.857150	21.4670
ERROR	12	1382.50000000	115.20833333		ROOT MSE		ACHIEVE MEAN
CORRECTED TOTAL	15	9678.00000000			10.73351449		50.00000000

SOURCE	DF	TYPE I SS	F VALUE	PR > F	DF	TYPE III SS	F VALUE	PR > F
IQM	1	8190.25000000	71.09	0.0001	1	8190.25000000	71.09	0.0001
SESM	1	49.00000000	0.43	0.5266	1	49.00000000	0.43	0.5266
IQM*SESM	1	56.25000000	0.49	0.4980	1	56.25000000	0.49	0.4980

ANOVA RESULTS - DEPENDENT VARIABLE: GPA

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: GPA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F	R-SQUARE	C.V.
MODEL	3	14.13687500	4.71229167	132.27	0.0001	0.970648	8.9084
ERROR	12	0.42750000	0.03562500		ROOT MSE		GPA MEAN
CORRECTED TOTAL	15	14.56437500			0.18874586		2.11875000

SOURCE	DF	TYPE I SS	F VALUE	PR > F	DF	TYPE III SS	F VALUE	PR > F
IQM	1	6.12562500	171.95	0.0001	1	6.12562500	171.95	0.0001
SESM	1	0.03062500	0.86	0.3721	1	0.03062500	0.86	0.3721
IQM*SESM	1	7.98062500	224.02	0.0001	1	7.98062500	224.02	0.0001

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GENERAL LINEAR MODELS PROCEDURE

MEANS

IQM	N	ACHIEVE	GPA
H	8	72.6250000	2.73750000
L	8	27.3750000	1.50000000

SESM	N	ACHIEVE	GPA
H	8	48.2500000	2.16250000
L	8	51.7500000	2.07500000

IQM	SESM	N	ACHIEVE	GPA
H	H	4	72.7500000	2.07500000
H	L	4	72.5000000	3.40000000
L	H	4	23.7500000	2.25000000
L	L	4	31.0000000	0.75000000

GENERAL LINEAR MODELS PROCEDURE

CHARACTERISTIC ROOTS AND VECTORS OF: E INVERSE * H, WHERE H = TYPE III SS&CP MATRIX FOR: IQM E = ERROR SS&CP MATRIX

CHARACTERISTIC ROOT	PERCENT	CHARACTERISTIC VECTOR V'EV=1	
		ACHIEVE	GPA
18.30104105	100.00	0.01262135	1.26696556
0.00000000	0.00	-0.02397193	0.87654953

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GENERAL LINEAR MODELS PROCEDURE

MANOVA TEST CRITERIA FOR THE HYPOTHESIS OF NO OVERALL IQM EFFECT

H = TYPE III SS&CP MATRIX FOR: IQM
E = ERROR SS&CP MATRIX
P = RANK OF (H+E) = 2
Q = HYPOTHESIS DF = 1
NE= DF OF E = 12
S = MIN(P,Q) = 1
M = .5(ABS(P-Q)-1) = 0.0
N = .5(NE-P) = 5.0

WILKS' CRITERION L = DET(E)/DET(H+E) = 0.05181068 (SEE RAO 1973 P 555)

EXACT F = (1-L)/L*(NE+Q-P)/P WITH P AND NE+Q-P DF

F(2,11) = 100.66 PROB > F = 0.0001

PILLAI'S TRACE V = TR(H*INV(H+E)) = 0.94818932 (SEE PILLAI'S TABLE #2)

F APPROXIMATION = (2N+S)/(2M+S+1) * V/(S-V) WITH S(2M+S+1) AND S(2N+S) DF

F(2,11) = 100.66 PROB > F = 0.0001

HOTELLING-LAWLEY TRACE = TR(E**-1*H) = 18.30104105 (SEE PILLAI'S TABLE #3)

F APPROXIMATION = (2S*N-S+2)*TR(E**-1*H)/(S*S*(2M+S+1)) WITH S(2M+S+1) AND 2S*N-S+2 DF

F(2,11) = 100.66 PROB > F = 0.0001

ROY'S MAXIMUM ROOT CRITERION = 18.30104105 (SEE AMS VOL 31 P 625)

FIRST CANONICAL VARIABLE YIELDS AN F UPPER BOUND

F(2,11) = 100.66 PROB > F = 0.0001

GENERAL LINEAR MODELS PROCEDURE

CHARACTERISTIC ROOTS AND VECTORS OF: E INVERSE * H, WHERE H = TYPE III SS&CP MATRIX FOR: SESM E = ERROR SS&CP MATRIX

CHARACTERISTIC ROOT	PERCENT	CHARACTERISTIC VECTOR V'EV=1	
		ACHIEVE	GPA
0.12095669	100.00	-0.01729925	1.29539177
0.00000000	0.00	0.02084918	0.83396701

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GENERAL LINEAR MODELS PROCEDURE

MANOVA TEST CRITERIA FOR THE HYPOTHESIS OF NO OVERALL SESM EFFECT

H = TYPE III SS&CP MATRIX FOR: SESM
E = ERROR SS&CP MATRIX
P = RANK OF (H+E) = 2
Q = HYPOTHESIS DF = 1
NE= DF OF E = 12
S = MIN(P,Q) = 1
M = .5(ABS(P-Q)-1) = 0.0
N = .5(NE-P) = 5.0

WILKS' CRITERION L = DET(E)/DET(H+E) = 0.89209513 (SEE RAO 1973 P 555)

EXACT F = (1-L)/L*(NE+Q-P)/P WITH P AND NE+Q-P DF

F(2,11) = 0.67 PROB > F = 0.5337

PILLAI'S TRACE V = TR(H*INV(H+E)) = 0.10790487 (SEE PILLAI'S TABLE #2)

F APPROXIMATION = (2N+S)/(2M+S+1) * V/(S-V) WITH S(2M+S+1) AND S(2N+S) DF

F(2,11) = 0.67 PROB > F = 0.5337

HOTELLING-LAWLEY TRACE = TR(E**-1*H) = 0.12095669 (SEE PILLAI'S TABLE #3)

F APPROXIMATION = (2S*N-S+2)*TR(E**-1*H)/(S*S*(2M+S+1)) WITH S(2M+S+1) AND 2S*N-S+2 DF

F(2,11) = 0.67 PROB > F = 0.5337

ROY'S MAXIMUM ROOT CRITERION = 0.12095669 (SEE AMS VOL 31 P 625)

FIRST CANONICAL VARIABLE YIELDS AN F UPPER BOUND

F(2,11) = 0.67 PROB > F = 0.5337

GENERAL LINEAR MODELS PROCEDURE

CHARACTERISTIC ROOTS AND VECTORS OF: E INVERSE * H, WHERE H = TYPE III SS&CP MATRIX FOR: IQM*SESM E = ERROR SS&CP MATRIX

CHARACTERISTIC ROOT	PERCENT	CHARACTERISTIC VECTOR V'EV=1	
		ACHIEVE	GPA
19.19642177	100.00	-0.00449429	1.53899644
0.00000000	0.00	0.02671616	0.07092787

GENERAL LINEAR MODELS PROCEDURE

MANOVA TEST CRITERIA FOR THE HYPOTHESIS OF NO OVERALL IQM*SESM EFFECT

H = TYPE III SS&CP MATRIX FOR: IQM*SESM
E = ERROR SS&CP MATRIX
P = RANK OF (H+E) = 2
Q = HYPOTHESIS DF = 1
NE= DF OF E = 12
S = MIN(P,Q) = 1
M = .5(ABS(P-Q)-1) = 0.0
N = .5(NE-P) = 5.0

WILKS' CRITERION L = DET(E)/DET(H+E) = 0.04951372 (SEE RAO 1973 P 555)

EXACT F = (1-L)/L*(NE+Q-P)/P WITH P AND NE+Q-P DF

F(2,11) = 105.58 PROB > F = 0.0001

PILLAI'S TRACE V = TR(H*INV(H+E)) = 0.95048628 (SEE PILLAI'S TABLE #2)

F APPROXIMATION = (2N+S)/(2M+S+1) * V/(S-V) WITH S(2M+S+1) AND S(2N+S) DF

F(2,11) = 105.58 PROB > F = 0.0001

HOTELLING-LAWLEY TRACE = TR(E**-1*H) = 19.19642177 (SEE PILLAI'S TABLE #3)

F APPROXIMATION = (2S*N-S+2)*TR(E**-1*H)/(S*S*(2M+S+1)) WITH S(2M+S+1) AND 2S*N-S+2 DF

F(2,11) = 105.58 PROB > F = 0.0001

ROY'S MAXIMUM ROOT CRITERION = 19.19642177 (SEE AMS VOL 31 P 625)

FIRST CANONICAL VARIABLE YIELDS AN F UPPER BOUND

F(2,11) = 105.58 PROB > F = 0.0001

CANONICAL CORRELATION ANALYSIS

IV's: IQ AND SES

16 OBSERVATIONS
2 'VAR' VARIABLES
2 'WITH' VARIABLES

	CANONICAL CORRELATION	ADJUSTED CANONICAL CORRELATION	APPROX STANDARD ERROR	SQUARED CANONICAL CORRELATION	EIGENVALUES OF INV(E)*H = CANRSQ/(1-CANRSQ) EIGENVALUE	DIFFERENCE	PROPORTION	CUMULATIVE
1	0.914198	0.907189	0.042411	0.835743	5.0880	5.0761	0.9977	0.9977
2	0.108818	.	0.255141	0.011841	0.0120	.	0.0023	1.0000

TESTS OF H0: THE CANONICAL CORRELATION IN THE CURRENT ROW AND ALL THAT FOLLOW ARE ZERO

	LIKELIHOOD RATIO	F	NUM DF	DEN DF	PR > F
1	0.16231157	8.8928	4	24	0.0001
2	0.98815859	0.1558	1	13	0.6995

MULTIVARIATE TEST STATISTICS AND F APPROXIMATIONS
S=2 M=-0.5 N=5.5

STATISTIC	VALUE	F	NUM DF	DEN DF	PR > F
WILKS' LAMBDA	0.1623116	8.893	4	24	0.0001
PILLAI'S TRACE	0.8475848	4.781	4	26	0.0050
HOTELLING-LAWLEY TRACE	5.100019	14.025	4	22	0.0001
ROY'S GREATEST ROOT	5.088035	33.072	2	13	0.0001

NOTE: F STATISTIC FOR ROY'S GREATEST ROOT IS AN UPPER BOUND
F STATISTIC FOR WILKS' LAMBDA IS EXACT

RAW CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1	V2
ZIQ	0.9982788491	0.0618362950
ZSES	-.0723192216	0.9974324231

RAW CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1	W2
ZACHIEVE	0.897088571	-0.785493915
ZGPA	0.170041344	1.179774895

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CANONICAL CORRELATION ANALYSIS

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1	V2
ZIQ	0.9981	0.0618
ZSES	-0.0723	0.9974

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1	W2
ZACHIEVE	0.8971	-0.7855
ZGPA	0.1701	1.1802

CANONICAL STRUCTURE

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THEIR CANONICAL VARIABLES

	V1	V2
ZIQ	0.9974	0.0723
ZSES	-0.0618	0.9981

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THEIR CANONICAL VARIABLES

	W1	W2
ZACHIEVE	0.9898	-0.1427
ZGPA	0.6588	0.7524

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THE CANONICAL VARIABLES OF THE 'WITH' VARIABLES

	W1	W2
ZIQ	0.9118	0.0079
ZSES	-0.0565	0.1086

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THE CANONICAL VARIABLES OF THE 'VAR' VARIABLES

	V1	V2
ZACHIEVE	0.9048	-0.0155
ZGPA	0.6022	0.0819

CANONICAL CORRELATION ANALYSIS

IV: PRODUCT OF IQ & SES

16 OBSERVATIONS
1 'VAR' VARIABLES
2 'WITH' VARIABLES

	CANONICAL CORRELATION	ADJUSTED CANONICAL CORRELATION	APPROX STANDARD ERROR	SQUARED CANONICAL CORRELATION	EIGENVALUES OF INV(E)*H = CANRSQ/(1-CANRSQ) EIGENVALUE	DIFFERENCE	PROPORTION	CUMULATIVE
1	0.824472	0.820326	0.082687	0.679755	2.1226	.	1.0000	1.0000

TESTS OF H0: THE CANONICAL CORRELATION IN THE CURRENT ROW AND ALL THAT FOLLOW ARE ZERO

	LIKELIHOOD RATIO	F	NUM DF	DEN DF	PR > F
1	0.32024546	13.7969	2	13	0.0006

MULTIVARIATE TEST STATISTICS AND EXACT F STATISTICS
S=1 M=0 N=6

STATISTIC	VALUE	F	NUM DF	DEN DF	PR > F
WILKS' LAMBDA	0.3202455	13.797	2	13	0.0006
PILLAI'S TRACE	0.6797545	13.797	2	13	0.0006
HOTELLING-LAWLEY TRACE	2.122605	13.797	2	13	0.0006
ROY'S GREATEST ROOT	2.122605	13.797	2	13	0.0006

RAW CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1
PROD	0.9869328972

RAW CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1
ZACHIEVE	0.740832524
ZGPA	-1.186660997

CANONICAL CORRELATION ANALYSIS

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1
PROD	1.0000

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1
ZACHIEVE	0.7409
ZGPA	-1.1871

CANONICAL STRUCTURE

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THEIR CANONICAL VARIABLES

	V1
PROD	1.0000

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THEIR CANONICAL VARIABLES

	W1
ZACHIEVE	0.0942
ZGPA	-0.7836

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THE CANONICAL VARIABLES OF THE 'WITH' VARIABLES

	W1
PROD	0.8245

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THE CANONICAL VARIABLES OF THE 'VAR' VARIABLES

	V1
ZACHIEVE	0.0777
ZGPA	-0.6460

CANONICAL CORRELATION ANALYSIS

IV's: IQ, SES, PRODUCT

16 OBSERVATIONS
3 'VAR' VARIABLES
2 'WITH' VARIABLES

	CANONICAL CORRELATION	ADJUSTED CANONICAL CORRELATION	APPROX STANDARD ERROR	SQUARED CANONICAL CORRELATION	EIGENVALUES OF INV(E)*H = CANRSQ/(1-CANRSQ) EIGENVALUE	DIFFERENCE	PROPORTION	CUMULATIVE
1	0.932722	0.917567	0.033573	0.869970	6.6906	4.2734	0.7346	0.7346
2	0.841046	.	0.075560	0.707358	2.4171	.	0.2654	1.0000

TESTS OF H0: THE CANONICAL CORRELATION IN THE CURRENT ROW AND ALL THAT FOLLOW ARE ZERO

	LIKELIHOOD RATIO	F	NUM DF	DEN DF	PR > F
1	0.03805213	15.1300	6	22	0.0001
2	0.29264219	14.5029	2	12	0.0006

MULTIVARIATE TEST STATISTICS AND F APPROXIMATIONS
S=2 M=0 N=5

STATISTIC	VALUE	F	NUM DF	DEN DF	PR > F
WILKS' LAMBDA	0.03805213	15.130	6	22	0.0001
PILLAI'S TRACE	1.577328	14.927	6	24	0.0001
HOTELLING-LAWLEY TRACE	9.107702	15.180	6	20	0.0001
ROY'S GREATEST ROOT	6.69056	26.762	3	12	0.0001

NOTE: F STATISTIC FOR ROY'S GREATEST ROOT IS AN UPPER BOUND
F STATISTIC FOR WILKS' LAMBDA IS EXACT

RAW CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1	V2
ZIQ	0.9108128419	0.4104961382
ZSES	0.0548672540	-.2856985170
PROD	-.4554678165	0.8807266379

RAW CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1	W2
ZACHIEVE	0.482389436	1.090444417
ZGPA	0.651501151	-0.998162869

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CANONICAL CORRELATION ANALYSIS

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1	V2
ZIQ	0.9107	0.4104
ZSES	0.0549	-0.2857
PROD	-0.4615	0.8924

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1	W2
ZACHIEVE	0.4824	1.0905
ZGPA	0.6517	-0.9985

CANONICAL STRUCTURE

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THEIR CANONICAL VARIABLES

	V1	V2
ZIQ	0.8900	0.4485
ZSES	-0.0059	-0.1454
PROD	-0.4112	0.8677

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THEIR CANONICAL VARIABLES

	W1	W2
ZACHIEVE	0.8374	0.5466
ZGPA	0.9145	-0.4046

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THE CANONICAL VARIABLES OF THE 'WITH' VARIABLES

	W1	W2
ZIQ	0.8301	0.3772
ZSES	-0.0055	-0.1223
PROD	-0.3836	0.7298

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THE CANONICAL VARIABLES OF THE 'VAR' VARIABLES

	V1	V2
ZACHIEVE	0.7811	0.4597
ZGPA	0.8530	-0.3403

CANONICAL CORRELATION ANALYSIS

IV's: IQ & SES

DV's: GPA ONLY

16 OBSERVATIONS
2 'VAR' VARIABLES
1 'WITH' VARIABLES

	CANONICAL CORRELATION	ADJUSTED CANONICAL CORRELATION	APPROX STANDARD ERROR	SQUARED CANONICAL CORRELATION	EIGENVALUES OF INV(E)*H = CANRSQ/(1-CANRSQ) EIGENVALUE	DIFFERENCE	PROPORTION	CUMULATIVE
1	0.607773	0.585962	0.162823	0.369388	0.5858	.	1.0000	1.0000

TESTS OF H0: THE CANONICAL CORRELATION IN THE CURRENT ROW AND ALL THAT FOLLOW ARE ZERO

	LIKELIHOOD RATIO	F	NUM DF	DEN DF	PR > F
1	0.63061229	3.8074	2	13	0.0499

MULTIVARIATE TEST STATISTICS AND EXACT F STATISTICS
S=1 M=0 N=6

STATISTIC	VALUE	F	NUM DF	DEN DF	PR > F
WILKS' LAMBDA	0.6306123	3.807	2	13	0.0499
PILLAI'S TRACE	0.3693877	3.807	2	13	0.0499
HOTELLING-LAWLEY TRACE	0.5857604	3.807	2	13	0.0499
ROY'S GREATEST ROOT	0.5857604	3.807	2	13	0.0499

RAW CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1
ZIQ	0.9975099933
ZSES	0.0626983340

RAW CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1
ZGPA	0.9996222949

CANONICAL CORRELATION ANALYSIS

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1
ZIQ	0.9974
ZSES	0.0627

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1
ZGPA	1.0000

CANONICAL STRUCTURE

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THEIR CANONICAL VARIABLES

	V1
ZIQ	0.9980
ZSES	0.0732

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THEIR CANONICAL VARIABLES

	W1
ZGPA	1.0000

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THE CANONICAL VARIABLES OF THE 'WITH' VARIABLES

	W1
ZIQ	0.6066
ZSES	0.0445

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THE CANONICAL VARIABLES OF THE 'VAR' VARIABLES

	V1
ZGPA	0.6078

CANONICAL CORRELATION ANALYSIS

16 OBSERVATIONS
1 'VAR' VARIABLES
1 'WITH' VARIABLES

IV: PRODUCT
DV: GPA

	CANONICAL CORRELATION	ADJUSTED CANONICAL CORRELATION	APPROX STANDARD ERROR	SQUARED CANONICAL CORRELATION	EIGENVALUES OF INV(E)*H = CANRSQ/(1-CANRSQ) EIGENVALUE	DIFFERENCE	PROPORTION	CUMULATIVE
1	0.646030	.	0.150438	0.417355	0.7163	.	1.0000	1.0000

TESTS OF H0: THE CANONICAL CORRELATION IN THE CURRENT ROW AND ALL THAT FOLLOW ARE ZERO

	LIKELIHOOD RATIO	F	NUM DF	DEN DF	PR > F
1	0.58264513	10.0283	1	14	0.0069

MULTIVARIATE TEST STATISTICS AND EXACT F STATISTICS
S=1 M=-0.5 N=6.5

STATISTIC	VALUE	F	NUM DF	DEN DF	PR > F
WILKS' LAMBDA	0.5826451	10.028	1	14	0.0069
PILLAI'S TRACE	0.4173549	10.028	1	14	0.0069
HOTELLING-LAWLEY TRACE	0.7163106	10.028	1	14	0.0069
ROY'S GREATEST ROOT	0.7163106	10.028	1	14	0.0069

RAW CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1
PROD	-.9869328972

RAW CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1
ZGPA	0.9996222949

CANONICAL CORRELATION ANALYSIS

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1
PROD	-1.0000

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1
ZGPA	1.0000

CANONICAL STRUCTURE

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THEIR CANONICAL VARIABLES

	V1
PROD	-1.0000

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THEIR CANONICAL VARIABLES

	W1
ZGPA	1.0000

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THE CANONICAL VARIABLES OF THE 'WITH' VARIABLES

	W1
PROD	-0.6460

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THE CANONICAL VARIABLES OF THE 'VAR' VARIABLES

	V1
ZGPA	0.6460

CANONICAL CORRELATION ANALYSIS
IV: IQ, SES, PRODUCT
DV: GPA

16 OBSERVATIONS
3 'VAR' VARIABLES
1 'WITH' VARIABLES

	CANONICAL CORRELATION	ADJUSTED CANONICAL CORRELATION	APPROX STANDARD ERROR	SQUARED CANONICAL CORRELATION	EIGENVALUE	EIGENVALUES OF INV(E)*H = CANRSQ/(1-CANRSQ) DIFFERENCE	PROPORTION	CUMULATIVE
1	0.918344	0.911768	0.040445	0.843356	5.3839	.	1.0000	1.0000

TESTS OF H0: THE CANONICAL CORRELATION IN THE CURRENT ROW AND ALL THAT FOLLOW ARE ZERO

	LIKELIHOOD RATIO	F	NUM DF	DEN DF	PR > F
1	0.15664421	21.5356	3	12	0.0001

MULTIVARIATE TEST STATISTICS AND EXACT F STATISTICS
S=1 M=0.5 N=5.5

STATISTIC	VALUE	F	NUM DF	DEN DF	PR > F
WILKS' LAMBDA	0.1566442	21.536	3	12	0.0001
PILLAI'S TRACE	0.8433558	21.536	3	12	0.0001
HOTELLING-LAWLEY TRACE	5.383894	21.536	3	12	0.0001
ROY'S GREATEST ROOT	5.383894	21.536	3	12	0.0001

RAW CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1
ZIQ	0.6938975947
ZSES	0.1568158753
PROD	-.7493680470

RAW CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1
ZGPA	0.9996222949

CANONICAL CORRELATION ANALYSIS

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1
ZIQ	0.6938
ZSES	0.1568
PROD	-0.7593

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1
ZGPA	1.0000

CANONICAL STRUCTURE

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THEIR CANONICAL VARIABLES

	V1
ZIQ	0.6605
ZSES	0.0484
PROD	-0.7035

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THEIR CANONICAL VARIABLES

	W1
ZGPA	1.0000

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THE CANONICAL VARIABLES OF THE 'WITH' VARIABLES

	W1
ZIQ	0.6066
ZSES	0.0445
PROD	-0.6460

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THE CANONICAL VARIABLES OF THE 'VAR' VARIABLES

	V1
ZGPA	0.9183

GROUP=1

CANONICAL CORRELATION ANALYSIS

8 OBSERVATIONS
3 'VAR' VARIABLES
2 'WITH' VARIABLES

	CANONICAL CORRELATION	ADJUSTED CANONICAL CORRELATION	APPROX STANDARD ERROR	SQUARED CANONICAL CORRELATION	EIGENVALUES OF INV(E)*H = CANRSQ/(1-CANRSQ) EIGENVALUE	DIFFERENCE	PROPORTION	CUMULATIVE
1	0.974020	0.964603	0.019384	0.948714	18.4985	15.0862	0.8443	0.8443
2	0.879410	.	0.085661	0.773362	3.4123	.	0.1557	1.0000

TESTS OF H0: THE CANONICAL CORRELATION IN THE CURRENT ROW AND ALL THAT FOLLOW ARE ZERO

	LIKELIHOOD RATIO	F	NUM DF	DEN DF	PR > F
1	0.01162332	8.2754	6	6	0.0106
2	0.22663778	6.8247	2	4	0.0514

MULTIVARIATE TEST STATISTICS AND F APPROXIMATIONS
S=2 M=0 N=1

STATISTIC	VALUE	F	NUM DF	DEN DF	PR > F
WILKS' LAMBDA	0.01162332	8.275	6	6	0.0106
PILLAI'S TRACE	1.722076	8.262	6	8	0.0044
HOTELLING-LAWLEY TRACE	21.91087	7.304	6	4	0.0374
ROY'S GREATEST ROOT	18.49855	24.665	3	4	0.0048

NOTE: F STATISTIC FOR ROY'S GREATEST ROOT IS AN UPPER BOUND
F STATISTIC FOR WILKS' LAMBDA IS EXACT

RAW CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1	V2
ZIQ	0.9630457669	0.2030855941
ZSES	-.0481960867	0.0800621395
PROD	0.2676084696	-.9847376918

GROUP=1

CANONICAL CORRELATION ANALYSIS

RAW CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1	W2
ZACHIEVE	0.849969895	-0.518950251
ZGPA	0.028183615	1.105246952

GROUP=1

CANONICAL CORRELATION ANALYSIS

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1	V2
ZIQ	0.9816	0.2069
ZSES	-0.0509	0.0845
PROD	0.2880	-0.9737

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1	W2
ZACHIEVE	0.9850	-0.6014
ZGPA	0.0294	1.1537

GROUP=1

CANONICAL STRUCTURE

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THEIR CANONICAL VARIABLES

	V1	V2
ZIQ	0.9592	0.2815
ZSES	0.0130	-0.0843
PROD	0.2050	-0.9745

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THEIR CANONICAL VARIABLES

	W1	W2
ZACHIEVE	0.9997	-0.0255
ZGPA	0.5211	0.8535

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THE CANONICAL VARIABLES OF THE 'WITH' VARIABLES

	W1	W2
ZIQ	0.9343	0.2476
ZSES	0.0127	-0.0742
PROD	0.1996	-0.8569

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THE CANONICAL VARIABLES OF THE 'VAR' VARIABLES

	V1	V2
ZACHIEVE	0.9737	-0.0224
ZGPA	0.5076	0.7506

GROUP=2

CANONICAL CORRELATION ANALYSIS

8 OBSERVATIONS
3 'VAR' VARIABLES
2 'WITH' VARIABLES

	CANONICAL CORRELATION	ADJUSTED CANONICAL CORRELATION	APPROX STANDARD ERROR	SQUARED CANONICAL CORRELATION	EIGENVALUES OF INV(E)*H = CANRSQ/(1-CANRSQ) EIGENVALUE	DIFFERENCE	PROPORTION	CUMULATIVE
1	0.975713	0.967312	0.018136	0.952016	19.8402	16.9320	0.8722	0.8722
2	0.862631	.	0.096709	0.744132	2.9083	.	0.1278	1.0000

TESTS OF H0: THE CANONICAL CORRELATION IN THE CURRENT ROW AND ALL THAT FOLLOW ARE ZERO

	LIKELIHOOD RATIO	F	NUM DF	DEN DF	PR > F
1	0.01227760	8.0249	6	6	0.0114
2	0.25586787	5.8165	2	4	0.0655

MULTIVARIATE TEST STATISTICS AND F APPROXIMATIONS
S=2 M=0 N=1

STATISTIC	VALUE	F	NUM DF	DEN DF	PR > F
WILKS' LAMBDA	0.0122776	8.025	6	6	0.0114
PILLAI'S TRACE	1.696148	7.443	6	8	0.0062
HOTELLING-LAWLEY TRACE	22.74849	7.583	6	4	0.0351
ROY'S GREATEST ROOT	19.84022	26.454	3	4	0.0042

NOTE: F STATISTIC FOR ROY'S GREATEST ROOT IS AN UPPER BOUND
F STATISTIC FOR WILKS' LAMBDA IS EXACT

RAW CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1	V2
ZIQ	0.8648777702	0.3791280194
ZSES	0.0469276050	-.5643100833
PROD	-.6052422136	0.7095143898

GROUP=2

CANONICAL CORRELATION ANALYSIS

RAW CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1	W2
ZACHIEVE	0.626212556	1.340989161
ZGPA	0.554278691	-1.116665217

GROUP=2

CANONICAL CORRELATION ANALYSIS

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'VAR' VARIABLES

	V1	V2
ZIQ	0.9020	0.3954
ZSES	0.0475	-0.5713
PROD	-0.6177	0.7241

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1	W2
ZACHIEVE	0.5412	1.1589
ZGPA	0.5687	-1.1457

GROUP=2

CANONICAL STRUCTURE

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THEIR CANONICAL VARIABLES

	V1	V2
ZIQ	0.7967	0.5181
ZSES	-0.0288	-0.4796
PROD	-0.4578	0.7197

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THEIR CANONICAL VARIABLES

	W1	W2
ZACHIEVE	0.8957	0.4446
ZGPA	0.9061	-0.4231

RELATIONS BETWEEN THE 'VAR' VARIABLES AND THE CANONICAL VARIABLES OF THE 'WITH' VARIABLES

	W1	W2
ZIQ	0.7773	0.4469
ZSES	-0.0281	-0.4137
PROD	-0.4467	0.6208

RELATIONS BETWEEN THE 'WITH' VARIABLES AND THE CANONICAL VARIABLES OF THE 'VAR' VARIABLES

	V1	V2
ZACHIEVE	0.8740	0.3835
ZGPA	0.8841	-0.3650

```
    *** OUTPUT FROM PROGRAM   RELA ***
   MODIFIED TO RUN ON THE VAX BY *BRUCE THOMPSON*
### invariance log

PARAMETERS
COL  1- 5 =     5
COL  6-10 =     2
COL 11-15 =     2
COL 16-20 =     0
COL 21-25 =     0

DATA FORMAT = (4X,2F14.4)


MATRIX A (TARGET):
  1 0.98160 0.20690
  2-0.05090 0.08450
  3 0.28800-0.97370
  4 0.98500-0.60140
  5 0.02940 1.15370


MATRIX B:
  1 0.90200 0.39540
  2 0.04750-0.57130
  3-0.61770 0.72410
  4 0.54120 1.15890
  5 0.56870-1.14570


PRINCIPAL AXIS ANALYSIS.

TRACE =   14.0438

100.00 PCT. OF TRACE WAS EXTRACTED BY  2 ROOTS.


COSINES AMONG FACTOR AXES
ROWS=A, COLUMNS=B

 A BY B          1          2

      1        0.7954     0.6061

      2        0.6061    -0.7954
ROWS=A, COLUMNS=B

 R TO Z          1          2

      1        1.0859     0.7028

      2        0.7028    -1.0859

 NEW B:          1          2

      1        0.9571     0.2322
```

2	-0.3085	0.4832
3	-0.0524	-0.9503
4	1.1329	-0.5937
5	-0.2421	1.2560

TEST R:	1	2	3	4	5
	0.9995	0.9997	0.9419	0.9979	0.9768

R TO Z	1	2	3	4	5
	3.8002	3.8002	1.7543	3.4196	2.2221

••• MEAN ITEM R = 0.98314

MEAN ITEM R-TO-Z = 2.99929